

SADLER MATHEMATICS

METHODS UNIT 3

WORKED SOLUTIONS

Chapter 1 Differentiation

Exercise 1A

Question 1

$$\frac{dy}{dx} = 5$$

Question 2

$$\frac{dy}{dx} = 6x - 2$$

Question 3

$$\frac{dy}{dx} = 6x^2 - 2x$$

Question 4

$$\frac{dy}{dx} = -2$$

Question 5

$$\frac{dy}{dx} = \frac{1}{5}$$

Question 6

$$y = \frac{5}{x}$$
$$= 5x^{-1}$$

$$\frac{dy}{dx} = -5x^{-2}$$
$$= -\frac{5}{x^2}$$

Question 7

$$y = 3x^2 - \frac{3}{x^2}$$
$$= 3x^2 - 3x^{-2}$$

$$\frac{dy}{dx} = 6x - (-2) \times 3x^{-3}$$
$$= 6x + \frac{6}{x^3}$$

Question 8

$$y = 10\sqrt{x}$$
$$= 10x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \times 10x^{-\frac{1}{2}}$$
$$= \frac{5}{\sqrt{x}}$$

Question 9

$$y = 10 + 4\sqrt{x}$$
$$= 10 + 4x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} - 4x^{-\frac{1}{2}}$$
$$= \frac{2}{\sqrt{x}}$$

Question 10

$$y = \frac{8}{\sqrt{x}}$$
$$= 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{-1}{2} \times 8x^{-\frac{3}{2}}$$
$$= -\frac{4}{x^{\frac{3}{2}}} \quad \text{or} \quad -\frac{4}{\sqrt{x^3}}$$

Question 11

$$y = \sqrt[3]{x}$$
$$= x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{3} \times x^{-\frac{2}{3}}$$
$$= \frac{1}{3x^{\frac{2}{3}}} \quad \text{or} \quad \frac{1}{3\sqrt[3]{x^2}}$$

Question 12

$$y = \frac{5x^2}{x} - \frac{8x}{x}$$
$$= 5x - 8$$

$$\frac{dy}{dx} = 5$$

Question 13

$$y = 6 + \frac{1}{x}$$
$$= 6 + x^{-1}$$

$$\frac{dy}{dx} = -1x^{-2}$$
$$= -\frac{1}{x^2}$$

Question 14

$$y = 35x^2 - 10$$

$$\frac{dy}{dx} = 70x$$

Question 15

$$y = 3x^3 - 3x + 2x^2 - 2$$

$$\frac{dy}{dx} = 9x^2 - 3 + 4x$$

$$= 9x^2 + 4x - 3$$

Question 16

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\frac{d^2y}{dx^2} = 2$$

Question 17

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 6x$$

Question 18

$$y = 3x^2 + x$$

$$\frac{dy}{dx} = 6x + 1$$

$$\frac{d^2y}{dx^2} = 6$$

Question 19

$$y = 2x^3 + 2x - 34$$

$$\frac{dy}{dx} = 6x^2 + 2$$

$$\frac{d^2y}{dx^2} = 12x$$

Question 20

$$y = 2x^2$$

$$\frac{dy}{dx} = 4x$$

$$\frac{d^2y}{dx^2} = 4$$

Question 21

$$y = 4x^3 + 3x^2 + 2x$$

$$\frac{dy}{dx} = 12x^2 + 6x + 2$$

$$\frac{d^2y}{dx^2} = 24x + 6$$

Question 22

$$y = \sqrt{x}$$

$$= x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \times \frac{1}{2} \times x^{-\frac{3}{2}}$$

$$= -\frac{1}{4x^{\frac{3}{2}}} \quad \text{or} \quad -\frac{1}{4\sqrt{x^3}}$$

Question 23

$$y = 8\sqrt{x}$$

$$= 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \times 8x^{-\frac{1}{2}}$$

$$= 4x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \times 4x^{-\frac{3}{2}}$$

$$= -\frac{2}{x^{\frac{3}{2}}} \quad \text{or} \quad -\frac{2}{\sqrt{x^3}}$$

Question 24

$$y = \frac{1}{x}$$

$$= x^{-1}$$

$$\frac{dy}{dx} = -1x^{-2}$$

$$\frac{d^2y}{dx^2} = (-2)(-1)x^{-3}$$

$$= \frac{2}{x^3}$$

Question 25

$$y = \frac{1}{5}x + 7$$

$$\frac{dy}{dx} = \frac{1}{5}$$

$$\frac{d^2y}{dx^2} = 0$$

Question 26

$$y = \frac{5}{x} + 7$$
$$= 5x^{-1} + 7$$

$$\frac{dy}{dx} = -5x^{-2}$$

$$\frac{d^2y}{dx^2} = (-2)(-5)x^{-3}$$
$$= \frac{10}{x^3}$$

Question 27

$$y = x^2 + \frac{4}{x^2}$$
$$= x^2 + 4x^{-2}$$

$$\frac{dy}{dx} = 2x + 8x^{-3}$$

$$\frac{d^2y}{dx^2} = 2 - (-3)(8)x^{-4}$$
$$= 2 + \frac{24}{x^4}$$

Question 28

$$f(x) = 3x - \frac{1}{x}$$
$$= 3x - x^{-1}$$

$$f'(x) = 3 - (-1)x^{-2}$$
$$= 3 + \frac{1}{x^2}$$

Question 29

$$f(x) = 5x^2 + 8\sqrt{x}$$
$$= 5x^2 + 8x^{\frac{1}{2}}$$

$$f'(x) = 10x + 4x^{-\frac{1}{2}}$$
$$= 10x + \frac{4}{\sqrt{x}}$$

Question 30

$$\begin{aligned}f(x) &= \frac{4x^2}{\sqrt{x}} \\ &= 4x^{\frac{3}{2}}\end{aligned}$$

$$\begin{aligned}f'(x) &= \frac{3}{2} \times 4x^{\frac{1}{2}} \\ &= 6\sqrt{x}\end{aligned}$$

Question 31

$$\begin{aligned}f(x) &= 3x^4 + 4x^3 \\ f'(x) &= 12x^3 + 12x^2 \\ f''(x) &= 36x^2 + 24x\end{aligned}$$

Question 32

$$\begin{aligned}f(x) &= \frac{3}{2x^3} \\ &= \frac{3}{2}x^{-3}\end{aligned}$$

$$\begin{aligned}f'(x) &= (-3) \times \frac{3}{2}x^{-4} \\ &= -\frac{9}{2}x^{-4}\end{aligned}$$

$$\begin{aligned}f''(x) &= (-4) \times -\frac{9}{2}x^{-5} \\ &= \frac{18}{x^5}\end{aligned}$$

Question 33

$$\begin{aligned}f(x) &= 5x^3 - \frac{1}{x^2} \\ &= 5x^3 - x^{-2}\end{aligned}$$

$$\begin{aligned}f'(x) &= 15x^2 - (-2)x^{-3} \\ &= 15x^2 + 2x^{-3}\end{aligned}$$

$$\begin{aligned}f''(x) &= 30x + (-3) \times 2x^{-3} \\ &= 30x - \frac{6}{x^3}\end{aligned}$$

Question 34

$$y = 2x^3 - 2x + 1$$

$$\frac{dy}{dx} = 6x^2 - 2$$

At $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= 6(1)^2 - 2 \\ &= 4\end{aligned}$$

Question 35

$$\begin{aligned}y &= 8 - \frac{5}{x} \\ &= 8 - 5x^{-1}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(-5)x^{-2} \\ &= \frac{5}{x^2}\end{aligned}$$

At $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{5}{(-1)^2} \\ &= 5\end{aligned}$$

Question 36

$$\begin{aligned}y &= 3x^2 - \frac{1}{x^2} \\ &= 3x^2 - x^{-2}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= 6x - (-2)x^{-3} \\ &= 6x + \frac{2}{x^3}\end{aligned}$$

At $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= 6(-1) + \frac{2}{(-1)^3} \\ &= -6 - 2 \\ &= -8\end{aligned}$$

Question 37

$$f(x) = 2x^3 - 3x^2 + 4x + 2$$

$$f'(x) = 6x^2 - 6x + 4$$

$$f''(x) = 12x - 6$$

$$\begin{aligned} f''(-3) &= 12(-3) - 6 \\ &= -42 \end{aligned}$$

Question 38

a $f'(x) = 5 - 6x^2$

b
$$\begin{aligned} f'(2) &= 5 - 6(2)^2 \\ &= 5 - 24 \\ &= -19 \end{aligned}$$

c $f''(x) = -12x$

d
$$\begin{aligned} f''(-2) &= -12(-2) \\ &= 24 \end{aligned}$$

Question 39

$$y = 5x^2$$

$$\frac{dy}{dx} = 10x$$

at $x = -2$,

$$\begin{aligned} \frac{dy}{dx} &= 10(-2) \\ &= -20 \end{aligned}$$

Equation of tangent

$$y = -20x + c$$

Using $(-2, 20)$

$$20 = -20(-2) + c$$

$$20 = 40 + c$$

$$c = -20$$

$$\therefore y = -20x - 20$$

Question 40

$$y = x + \frac{6}{x}$$
$$= x + 6x^{-1}$$

$$\frac{dy}{dx} = 1 - (-1) \times 6x^{-2}$$
$$= 1 - \frac{6}{x^2}$$

At $x = 2$,

$$\frac{dy}{dx} = -1 - \frac{6}{2^2}$$
$$= -0.5$$

Equation of tangent

$$y = -0.5x + c$$

Using (2, 5)

$$5 = -0.5(2) + c$$

$$5 = -1 + c$$

$$6 = c$$

$$\therefore y = -0.5x + 6$$

Question 41

$$y = \frac{x^3}{x} + \frac{2x^{\frac{1}{2}}}{x}$$

$$= x^2 + 2x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 2x + \left(-\frac{1}{2}\right) \times 2x^{-\frac{3}{2}}$$

$$= 2x - \frac{1}{\sqrt{x^3}}$$

At $x = 1$,

$$\frac{dy}{dx} = 2(1) - \frac{1}{\sqrt{1^3}}$$

$$= 1$$

Equation of tangent

$$y = 1x + c$$

Using (1, 3)

$$3 = 1(1) + c$$

$$c = 2$$

$$\therefore y = x + 2$$

Question 42

a $y = 2x^3 + 6x^2 - 8x + 4$

$$\frac{dy}{dx} = 6x^2 + 12x - 8$$

$$10 = 6x^2 + 12x - 8$$

$$0 = 6x^2 + 12x - 18$$

$$0 = x^2 + 2x - 3$$

$$0 = (x+3)(x-1)$$

$$\therefore x = -3, 1$$

at $x = -3$,

$$\begin{aligned} y &= 2(-3)^3 + 6(-3)^2 - 8(-3) + 4 \\ &= 28 \end{aligned}$$

At $x = 1$

$$\begin{aligned} y &= 2(1)^3 + 6(1)^2 - 8(1) + 4 \\ &= 4 \end{aligned}$$

\therefore at $(-3, 28)$ and $(1, 4)$

b $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$

$$5 = \frac{3}{\sqrt{x}}$$

$$\sqrt{x} = \frac{3}{5}$$

$$x = \frac{9}{25}$$

$$= 0.36$$

At $x = 0.36$

$$\begin{aligned} y &= 5 + 6\sqrt{0.36} \\ &= 8.6 \end{aligned}$$

$\therefore (0.36, 8.6)$

Question 43

a
$$y = \frac{x^3}{12}$$
$$\frac{dy}{dx} = \frac{3x^2}{12}$$
$$= \frac{x^2}{4}$$

$$\frac{d^2y}{dx^2} = \frac{1}{4} \times 2x$$
$$\frac{3}{2} = \frac{x}{2}$$
$$x = 3$$

At $x = 3$,

$$y = \frac{3^3}{12}$$
$$= 2.25$$

$\therefore (3, 2.25)$

b
$$y = x^3 - 2x^2$$
$$\frac{dy}{dx} = 3x^2 - 4x$$
$$\frac{d^2y}{dx^2} = 6x - 4$$
$$2 = 6x - 4$$
$$6 = 6x$$
$$x = 1$$

At $x = 1$,

$$y = 1^3 - 2(1)^2$$
$$= -1$$

$\therefore (1, -1)$

Question 44

Using $(-1, 4)$

$$4 = a(-1)^3 + b(-1)^2 + c(-1) + 5$$

$$-1 = -1 + 2 - c \quad \rightarrow \quad \text{Equation 1}$$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

At $x = -1$,

$$\frac{dy}{dx} = 3a(-1)^2 + 2b(-1) + c$$

$$8 = 3a - 2b + c \quad \rightarrow \quad \text{Equation 2}$$

$$\frac{d^2y}{dx^2} = 6ax + 2b$$

At $x = -1$,

$$\frac{d^2y}{dx^2} = 6a(-1) + 2b$$

$$-24 = -6a + 2b$$

$$-12 = -3a + b \quad \rightarrow \quad \text{Equation 3}$$

By ClassPad, solving Equation 1, 2 and 3 simultaneously gives

$$a = 5, b = 3, c = -1$$

Exercise 1B

Question 1

$$y = (x)(x^2)$$

$$\begin{aligned}\frac{dy}{dx} &= x(2x) + x^2 \times 1 \\ &= 3x^2\end{aligned}$$

Question 2

$$y = (x+6)(x+1)$$

$$\begin{aligned}\frac{dy}{dx} &= (x+6) \times 1 + (x+1) \times 1 \\ &= 2x+7\end{aligned}$$

Question 3

$$y = (x+7)(x-3)$$

$$\begin{aligned}\frac{dy}{dx} &= (x+7) \times 1 + (x-3) \times 1 \\ &= 2x+4\end{aligned}$$

Question 4

$$y = (3x+1)(x+4)$$

$$\begin{aligned}\frac{dy}{dx} &= (3x+1) \times 1 + (x+4) \times 3 \\ &= 6x+13\end{aligned}$$

Question 5

$$y = (x+1)(3x+4)$$

$$\begin{aligned}\frac{dy}{dx} &= (x+1) \times 3 + (3x+4) \times 1 \\ &= 3x+3+3x+4 \\ &= 6x+7\end{aligned}$$

Question 6

$$y = (2x + 3)(5x + 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (2x + 3) \times 5 + (5x + 1) \times 2 \\ &= 10x + 15 + 10x + 2 \\ &= 20x + 17\end{aligned}$$

Question 7

$$y = (6x + 5)(2x + 3)$$

$$\begin{aligned}\frac{dy}{dx} &= (6x + 5) \times 2 + (2x + 3) \times 6 \\ &= 12x + 10 + 12x + 18 \\ &= 24x + 28\end{aligned}$$

Question 8

$$y = (x + 4)(x^2 + 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (x + 4) \times 2x + (x^2 + 2) \times 1 \\ &= 2x^2 + 8x + x^2 + 2 \\ &= 3x^2 + 8x + 2\end{aligned}$$

Question 9

$$y = (x + 5)(x^2 - 3)$$

$$\begin{aligned}\frac{dy}{dx} &= (x + 5) \times 2x + (x^2 - 3) \times 1 \\ &= 2x^2 + 10x + x^2 - 3 \\ &= 3x^2 + 10x - 3\end{aligned}$$

Question 10

$$y = (x + 7)(x^2 + 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (x + 7) \times 2x + (x^2 + 1) \times 1 \\ &= 2x^2 + 14x + x^2 + 1 \\ &= 3x^2 + 14x + 1\end{aligned}$$

Question 11

$$y = (x-10)(x^2 + 8)$$

$$\begin{aligned}\frac{dy}{dx} &= (x-10) \times 2x + (x^2 + 8) \times 1 \\ &= 2x^2 - 20x + x^2 + 8 \\ &= 3x^2 - 20x + 8\end{aligned}$$

Question 12

$$y = (2x-1)(x^2 + 7x - 2)$$

$$\begin{aligned}\frac{dy}{dx} &= (2x-1)(2x+7) + (x^2 + 7x - 2) \times 2 \\ &= 4x^2 + 14x - 2x - 7 + 2x^2 + 14x - 4 \\ &= 6x^2 + 26x - 11\end{aligned}$$

Question 13

$$y = (3x+4)(x^2 - 3x + 4)$$

$$\begin{aligned}\frac{dy}{dx} &= (3x+4)(2x-3) + (x^2 - 3x + 4) \times 3 \\ &= 6x^2 - 9x + 8x - 12 + 3x^2 - 9x + 12 \\ &= 9x^2 - 10x\end{aligned}$$

Question 14

$$y = (2x-3)(x^2 + 5x - 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (2x-3)(2x+5) + (x^2 + 5x - 1) \times 2 \\ &= 4x^2 + 10x - 6x - 15 + 2x^2 + 10x - 2 \\ &= 6x^2 + 14x - 17\end{aligned}$$

Question 15

$$y = (3x+1)(x^2 - 7x + 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (3x+1)(2x-7) + (x^2 - 7x + 1) \times 3 \\ &= 6x^2 - 21x + 2x - 7 + 3x^2 - 21x + 3 \\ &= 9x^2 - 40x - 4\end{aligned}$$

Question 16

$$y = (x+3)(x-2)$$

$$\frac{dy}{dx} = (x+3) \times 1 + (x-2) \times 1$$

$$= x+3+x-2$$

$$= 2x+1$$

At $x = 3$,

$$\frac{dy}{dx} = 2(3)+1$$

$$= 7$$

Question 17

$$y = (3x+1)(x-5)$$

$$\frac{dy}{dx} = (3x+1) \times 1 + (x-5) \times 3$$

$$= 3x+1+3x-15$$

$$= 6x-14$$

At $x = 3$,

$$\frac{dy}{dx} = 6(3)-14$$

$$= 4$$

Question 18

$$y = (3x-2)(2x+1)$$

$$\frac{dy}{dx} = (3x-2) \times 2 + (2x+1) \times 3$$

$$= 6x-4+6x+3$$

$$= 12x-1$$

At $x = 1$,

$$\frac{dy}{dx} = 12(1)-1$$

$$= 11$$

Question 19

$$y = (x-4)(x^2 - 1)$$

$$\frac{dy}{dx} = (x-4) \times 2x + (x^2 - 1) \times 1$$

$$= 2x^2 - 8x + x^2 - 1$$

$$= 3x^2 - 8x - 1$$

At $x = 2$,

$$\frac{dy}{dx} = 3(2)^2 - 8(2) - 1$$

$$= -5$$

Question 20

$$y = (3x-5)(x+2)$$

$$\frac{dy}{dx} = (3x-5) \times 1 + (x+2) \times 3$$

$$= 3x - 5 + 3x + 6$$

$$= 6x + 1$$

At $x = 2$,

$$\frac{dy}{dx} = 6(2) + 1$$

$$= 13$$

$$4 = 13(2) + c$$

$$4 = 26 + c$$

$$-22 = c$$

$$\therefore y = 13x - 22$$

Question 21

$$y = (1 + 2x)(5x - 1)$$

$$\begin{aligned}\frac{dy}{dx} &= (1 + 2x) \times 5 + (5x - 1) \times 2 \\ &= 5 + 10x + 10x - 2 \\ &= 20x + 3\end{aligned}$$

At $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= 20(1) + 3 \\ &= 23\end{aligned}$$

$$12 = 23(1) + c$$

$$12 = 23 + c$$

$$-11 = c$$

$$\therefore y = 23x - 11$$

Question 22

$$y = (2x - 1)(3x + 4)$$

$$\begin{aligned}\frac{dy}{dx} &= (2x - 1) \times 3 + (3x + 4) \times 2 \\ &= 6x - 3 + 6x + 8 \\ &= 12x + 5\end{aligned}$$

$$12x + 5 = -1$$

$$12x = -6$$

$$x = -\frac{1}{2}$$

$$\therefore \left(-\frac{1}{2}, -5\right)$$

Question 23

$$y = (x-3)(2x^2 - 11)$$

$$\frac{dy}{dx} = (x-3) \times 4x + (2x^2 - 11) \times 1$$

$$= 4x^2 - 12x + 2x^2 - 11$$

$$= 6x^2 - 12x - 11$$

$$6x^2 - 12x - 11 = 37$$

$$6x^2 - 12x - 48 = 0$$

$$6(x^2 + 2x - 8) = 0$$

$$6(x-4)(x+2) = 0$$

$$\therefore x = -2, 4$$

$$\text{At } x = -2,$$

$$y = (-2-3)(8-11)$$

$$= -5 \times (-3)$$

$$= 15$$

$$\therefore (-2, 15)$$

$$\text{At } x = 4,$$

$$y = (4-3)(32-11)$$

$$= 1 \times (21)$$

$$= 21$$

$$\therefore (4, 21)$$

Question 24

$$y = (x-3)(x^2 - 8)$$

$$\frac{dy}{dx} = (x-3) \times 2x + (x^2 - 8) \times 1$$

$$= 2x^2 - 6x + x^2 - 8$$

$$= 3x^2 - 6x - 8$$

$$y = x, y' = 1$$

$$3x^2 - 6x - 8 = 1$$

$$3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$3(x-3)(x+1) = 0$$

$$\therefore x = -1, 3$$

$$\text{At } x = -1,$$

$$y = (-1-3)(1-8)$$

$$= -4 \times (-7)$$

$$= 28$$

$$\therefore (-1, 28)$$

$$\text{At } x = 3,$$

$$y = (3-3)(3-8)$$

$$= 0 \times (-5)$$

$$= 0$$

$$\therefore (3, 0)$$

Question 25

a

$$\begin{aligned}y &= \sqrt{x^3} \times (2x+1) \\ &= x^{\frac{3}{2}} \times (2x+1) \\ \frac{dy}{dx} &= x^{\frac{3}{2}} \times 2 + (2x+1) \times \frac{3}{2} x^{\frac{1}{2}} \\ &= 2x^{\frac{3}{2}} + \frac{3}{2} \times 2x^{\frac{3}{2}} + \frac{3}{2} x^{\frac{1}{2}} \\ &= 2x^{\frac{3}{2}} + 3x^{\frac{3}{2}} + \frac{3}{2} x^{\frac{1}{2}} \\ &= 5x^{\frac{3}{2}} + \frac{3x^{\frac{1}{2}}}{2} \quad \text{or} \quad 5\sqrt{x^3} + \frac{3}{2}\sqrt{x}\end{aligned}$$

b

$$\begin{aligned}y &= \sqrt{x^3} (2x+1) \\ &= x^{\frac{3}{2}} (2x^1 + 1) \\ &= 2x^{\frac{5}{2}} + x^{\frac{3}{2}} \\ \frac{dy}{dx} &= 2 \times \frac{5}{2} x^{\frac{3}{2}} + \frac{3}{2} x^{\frac{1}{2}} \\ &= 5x^{\frac{3}{2}} + \frac{3x^{\frac{1}{2}}}{2} \quad \text{or} \quad 5\sqrt{x^3} + \frac{3}{2}\sqrt{x}\end{aligned}$$

Exercise 1C

Question 1

$$\begin{aligned}y &= \frac{x^5}{x^3} \\ \frac{dy}{dx} &= \frac{x^3 \times 5x^4 - x^5 \times 3x^2}{x^6} \\ &= \frac{5x^7 - 3x^7}{x^6} \\ &= \frac{2x^7}{x^6} \\ &= 2x\end{aligned}$$

Question 2

$$\begin{aligned}y &= \frac{1}{x^n} \\ \frac{dy}{dx} &= \frac{x^n \times 0 - 1nx^{n-1}}{x^{2n}} \\ &= \frac{-nx^{n-1}}{x^{2n}} \\ &= \frac{-n}{x^{n+1}} \quad \text{or} \quad -nx^{-n-1}\end{aligned}$$

Question 3

$$\begin{aligned}y &= \frac{2x}{x+3} \\ \frac{dy}{dx} &= \frac{(x+3) \times 2 - 2x \times 1}{(x+3)^2} \\ &= \frac{2x+6-2x}{(x+3)^2} \\ &= \frac{6}{(x+3)^2}\end{aligned}$$

Question 4

$$\begin{aligned}y &= \frac{3x}{5x-1} \\ \frac{dy}{dx} &= \frac{(5x-1) \times 3 - 3x \times 5}{(5x-1)^2} \\ &= \frac{15x-3-15x}{(5x-1)^2} \\ &= -\frac{3}{(5x-1)^2}\end{aligned}$$

Question 5

$$\begin{aligned}y &= \frac{6x}{4x-3} \\ \frac{dy}{dx} &= \frac{(4x-3) \times 6 - 6x \times 4}{(4x-3)^2} \\ &= \frac{24x-18-24x}{(4x-3)^2} \\ &= -\frac{18}{(4x-3)^2}\end{aligned}$$

Question 6

$$\begin{aligned}y &= \frac{7x}{1-2x} \\ \frac{dy}{dx} &= \frac{(1-2x) \times 7 - 7x \times (-2)}{(1-2x)^2} \\ &= \frac{7-14x+14x}{(1-2x)^2} \\ &= \frac{7}{(1-2x)^2}\end{aligned}$$

Question 7

$$\begin{aligned}y &= \frac{5x+1}{2x+3} \\ \frac{dy}{dx} &= \frac{(2x+3) \times 5 - (5x+1) \times 2}{(2x+3)^2} \\ &= \frac{10x+15-10x-2}{(2x+3)^2} \\ &= \frac{13}{(2x+3)^2}\end{aligned}$$

Question 8

$$\begin{aligned}y &= \frac{5x+1}{2x-3} \\ \frac{dy}{dx} &= \frac{(2x-3) \times 5 - (5x+1) \times 2}{(2x-3)^2} \\ &= \frac{10x-15-10x-2}{(2x-3)^2} \\ &= -\frac{17}{(2x-3)^2}\end{aligned}$$

Question 9

$$\begin{aligned}y &= \frac{6x-1}{5x+2} \\ \frac{dy}{dx} &= \frac{(5x+2) \times 6 - (6x-1) \times 5}{(5x+2)^2} \\ &= \frac{30x+12-30x+5}{(5x+2)^2} \\ &= \frac{17}{(5x+2)^2}\end{aligned}$$

Question 10

$$\begin{aligned}y &= \frac{3x-1}{2x-1} \\ \frac{dy}{dx} &= \frac{(2x-1) \times 3 - (3x-1) \times 2}{(2x-1)^2} \\ &= \frac{6x-3-6x+2}{(2x-1)^2} \\ &= -\frac{1}{(2x-1)^2}\end{aligned}$$

Question 11

$$\begin{aligned}y &= \frac{1-3x}{3x+1} \\ \frac{dy}{dx} &= \frac{(3x+1)(-3) - (1-3x) \times 3}{(3x+1)^2} \\ &= \frac{-9x-3-3+9x}{(3x+1)^2} \\ &= -\frac{6}{(3x+1)^2}\end{aligned}$$

Question 12

$$\begin{aligned}y &= \frac{5x}{x^2+1} \\ \frac{dy}{dx} &= \frac{(x^2+1) \times 5 - (5x)(2x)}{(x^2+1)^2} \\ &= \frac{5x^2+5-10x^2}{(x^2+1)^2} \\ &= \frac{-5x^2+5}{(x^2+1)^2} \\ &= \frac{5(1-x^2)}{(x^2+1)^2}\end{aligned}$$

Question 13

$$\begin{aligned}y &= \frac{2x^2}{x^3 + 1} \\ \frac{dy}{dx} &= \frac{(x^3 + 1)(4x) - (2x^2)(3x^2)}{(x^3 + 1)^2} \\ &= \frac{4x^4 + 4x - 6x^4}{(x^3 + 1)^2} \\ &= \frac{-2x^4 + 4x}{(x^3 + 1)^2} \\ &= \frac{2x(2 - x^3)}{(x^3 + 1)^2}\end{aligned}$$

Question 14

$$\begin{aligned}y &= \frac{3x^2}{x^5 + 3} \\ \frac{dy}{dx} &= \frac{(x^5 + 3)(6x) - (3x^2)(5x^4)}{(x^5 + 3)^2} \\ &= \frac{6x^6 + 18x - 15x^6}{(x^5 + 3)^2} \\ &= \frac{-9x^6 + 18x}{(x^5 + 3)^2} \\ &= \frac{9x(2 - x^5)}{(x^5 + 3)^2}\end{aligned}$$

Question 15

$$\begin{aligned}y &= \frac{3x}{6x-2} \\ \frac{dy}{dx} &= \frac{(x-2) \times 3 - 3x \times 1}{(6x-2)^2} \\ &= \frac{3x-6-3x}{(6x-2)^2} \\ &= \frac{-6}{(6x-2)^2}\end{aligned}$$

At $x = 4$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{-6}{(4-2)^2} \\ &= \frac{-6}{(2)^2} \\ &= \frac{-3}{2} \quad \text{or} \quad -1.5\end{aligned}$$

Question 16

$$\begin{aligned}y &= \frac{4x}{x^2-1} \\ \frac{dy}{dx} &= \frac{(x^2-1) \times 4 - (4x)(2x)}{(x^2-1)^2} \\ &= \frac{4x^2-4-8x^2}{(x^2-1)^2} \\ &= \frac{-4x^2-4}{(x^2-1)^2}\end{aligned}$$

At $x = 3$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{-4(3)^2-4}{(3^2-1)^2} \\ &= \frac{-40}{64} \\ &= -\frac{5}{8} \quad \text{or} \quad -0.625\end{aligned}$$

Question 17

$$y = \frac{3x+5}{x-3}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x-3) \times 3 - (3x+5) \times 1}{(x-3)^2} \\ &= \frac{3x-9-3x-5}{(x-3)^2} \\ &= \frac{-14}{(x-3)^2}\end{aligned}$$

At $x = 5$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{-14}{(5-3)^2} \\ &= \frac{-14}{(2)^2} \\ &= -\frac{7}{2} \quad \text{or} \quad -3.5\end{aligned}$$

$$10 = -3.5(5) + c$$

$$10 = -17.5 + c$$

$$c = 27.5$$

$$\therefore y = -3.5x + 27.5$$

Question 18

$$y = \frac{2x-1}{5-4x}$$

$$\frac{dy}{dx} = \frac{(5-4x) \times 2 - (2x-1)(-4)}{(5-4x)^2}$$

$$= \frac{10-8x+8x-4}{(5-4x)^2}$$

$$= \frac{6}{(5-4x)^2}$$

$$\frac{6}{(5-4x)^2} = 6$$

$$(5-4x)^2 = 1$$

$$5-4x=1 \quad \text{or} \quad 5-4x=-1$$

$$-4x = -4$$

$$-4x = -6$$

$$x = 1$$

$$x = 1.5$$

At $x = 1$

At $x = 1.5$

$$y = \frac{2(1)-1}{5-4(1)}$$

$$y = \frac{2(1.5)-1}{5-4(1.5)}$$

$$= \frac{1}{1}$$

$$= \frac{2}{-1}$$

$\therefore (1, 1)$

$\therefore (1.5, -2)$

Question 19

a $y = \frac{2x-3}{x}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{x \times 2 - (2x-3) \times 1}{x^2} \\ &= \frac{2x - 2x + 3}{x^2} \\ &= \frac{3}{x^2}\end{aligned}$$

b $y = (2x-3)x^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (2x-3)(-x^{-2}) + x^{-1} \times 2 \\ &= \frac{-(2x-3)}{x^2} + \frac{2}{x} \\ &= \frac{-2x+3+2x}{x^2} \\ &= \frac{3}{x^2}\end{aligned}$$

c $y = 2 - \frac{3}{x} = 2 - 3x^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= -(-1)3x^{-2} \\ &= \frac{3}{x^2}\end{aligned}$$

Exercise 1D

Question 1

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 7(4x+5)\end{aligned}$$

Question 2

$$\begin{aligned}\frac{dp}{dt} &= \frac{dp}{ds} \times \frac{ds}{dt} \\ &= 6s \times 2 \\ &= 12(2t+1)\end{aligned}$$

Question 3

$$\begin{aligned}\frac{dh}{dr} &= \frac{dh}{dp} \times \frac{dp}{dr} \\ &= 10p \times (-4r) \\ &= -40r(1-2r^2) \\ &= 40r(2r^2-1)\end{aligned}$$

Question 4

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dp} \times \frac{dp}{dx} \\ &= 2u \times 4 \times 3 \\ &= 24u \\ &= 24(4p-3) \\ &= 24(4(3x+2)-3) \\ &= 24(12x+8-3) \\ &= 24(12x+5)\end{aligned}$$

Question 5

$$\begin{aligned}y &= u^5, u = 3x+2 \\ \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 5u^4 \times 3 \\ &= 15(3x+2)^4\end{aligned}$$

Question 6

$$y = u^3, u = x^2 + 2$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 3u^2 \times 2x \\ &= 6x(x^2 + 2)^2\end{aligned}$$

Question 7

$$y = u^{-1}, u = 8x - 3$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -1u^{-2} \times 8 \\ &= \frac{-8}{(8x - 3)^2}\end{aligned}$$

Question 8

$$y = u^{\frac{1}{2}}, u = 2x + 3$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \times 2 \\ &= (2x + 3)^{-\frac{1}{2}} \\ &= \frac{1}{\sqrt{2x + 3}}\end{aligned}$$

Question 9

$$y = u^{-\frac{1}{2}}, u = 6x + 1$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\frac{1}{2}u^{-\frac{3}{2}} \times 6 \\ &= \frac{-3}{\sqrt{(6x + 1)^3}}\end{aligned}$$

Question 10

$$y = u^{-2}, u = 3x^2 + 2x + 1$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -2u^{-3} \times (6x + 2) \\ &= -\frac{2(6x + 2)}{(3x^2 + 2x + 1)^3} \\ &= -\frac{4(3x + 1)}{(3x^2 + 2x + 1)^3}\end{aligned}$$

Question 11

$$\begin{aligned}\frac{dy}{dx} &= 4(5x + 2)^3 \cdot 5 \\ &= 20(5x + 2)^3\end{aligned}$$

Question 12

$$\begin{aligned}\frac{dy}{dx} &= 3(7x - 3)^2 \cdot 7 \\ &= 21(7x - 3)^2\end{aligned}$$

Question 13

$$\begin{aligned}\frac{dy}{dx} &= 3(2 - 3x)^2 \cdot (-3) \\ &= -9(2 - 3x)^2\end{aligned}$$

Question 14

$$\begin{aligned}\frac{dy}{dx} &= 2(4 + 7x) \cdot 7 \\ &= 14(4 + 7x)\end{aligned}$$

Question 15

$$\begin{aligned}\frac{dy}{dx} &= 3(3x^2 + 5)^2 \cdot 6x \\ &= 18x(3x^2 + 5)^2\end{aligned}$$

Question 16

$$\begin{aligned}\frac{dy}{dx} &= 6(2x^3 + 1)^5 \cdot 6x^2 \\ &= 36x^2(2x^3 + 1)^5\end{aligned}$$

Question 17

$$\begin{aligned}\frac{dy}{dx} &= -3(x + 2)^{-4} \cdot 1 \\ &= -\frac{3}{(x + 2)^4}\end{aligned}$$

Question 18

$$\begin{aligned}\frac{dy}{dx} &= -1(2x + 5)^{-2} \cdot 2 \\ &= -\frac{2}{(2x + 5)^2}\end{aligned}$$

Question 19

$$\begin{aligned}\frac{dy}{dx} &= -1(x + 2)^{-2} \\ &= -\frac{1}{(x + 2)^2}\end{aligned}$$

Question 20

$$\begin{aligned}\frac{dy}{dx} &= -2(7x - 3)^{-3} \cdot 7 \\ &= -\frac{14}{(7x - 3)^3}\end{aligned}$$

Question 21

$$\begin{aligned}\frac{dy}{dx} &= 3 + 5(2x + 3)^4 \cdot 2 \\ &= 10(2x + 3)^4 + 3\end{aligned}$$

Question 22

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1 \\ &= \frac{1}{2\sqrt{x+1}}\end{aligned}$$

Question 23

$$\begin{aligned}\frac{dy}{dx} &= 5(10x+1)^4 \cdot 10 \\ &= 50(10x+1)^4\end{aligned}$$

When $x = 0$,

$$\begin{aligned}\frac{dy}{dx} &= 50(1)^4 \\ &= 50\end{aligned}$$

Question 24

$$\begin{aligned}\frac{dy}{dx} &= 3(6x-1)^2 \cdot 6 \\ &= 18(6x-1)^2\end{aligned}$$

When $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= 18(5)^2 \\ &= 450\end{aligned}$$

Question 25

$$\begin{aligned}\frac{dy}{dx} &= 3(1+x^4)^2 \cdot 4x^3 \\ &= 12x^3(1+x^4)^2\end{aligned}$$

When $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= 12 \cdot (-1)^3 \cdot 2^2 \\ &= -48\end{aligned}$$

Question 26

$$\begin{aligned}\frac{dy}{dx} &= -4(2x-3)^{-5} \cdot 2 \\ &= -8(2x-3)^{-5}\end{aligned}$$

When $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= -8(1)^{-5} \\ &= -8\end{aligned}$$

Question 27

$$\begin{aligned}\frac{dy}{dx} &= -3(2x^2+1)^{-4} \cdot 4x \\ &= -\frac{12x}{(2x^2+1)^4}\end{aligned}$$

When $x = 0$,

$$\begin{aligned}\frac{dy}{dx} &= -\frac{0}{1^4} \\ &= 0\end{aligned}$$

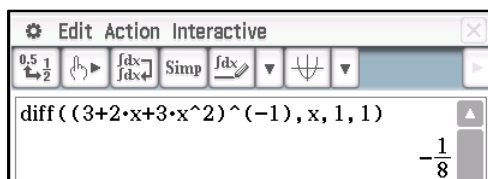
Question 28

$$\begin{aligned}\frac{dy}{dx} &= 2x + 5(x-1)^4 \cdot 1 \\ &= 2x + 5(x-1)^4\end{aligned}$$

When $x = 2$,

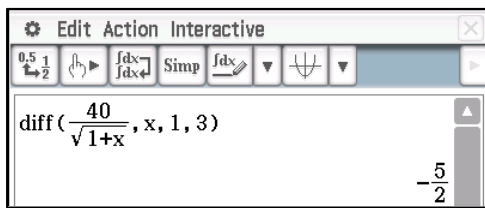
$$\begin{aligned}\frac{dy}{dx} &= 4 + 5(1)^4 \\ &= 9\end{aligned}$$

Question 29



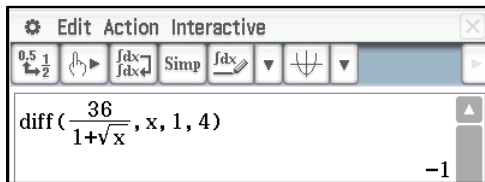
The screenshot shows a window titled "Edit Action Interactive" with a toolbar containing icons for undo, redo, differentiation (d/dx), simplification (Simp), and integration (∫). The input field contains the expression $\text{diff}((3+2 \cdot x+3 \cdot x^2)^{-1}, x, 1, 1)$ and the output field displays the result $-\frac{1}{8}$.

Question 30



A screenshot of a CAS interface window titled "Edit Action Interactive". The window contains a toolbar with icons for "0.5 1/2", a hand, "f dx / f dx d", "Simp", "f dx", and a plus sign. Below the toolbar, the input field contains the expression $\text{diff}\left(\frac{40}{\sqrt{1+x}}, x, 1, 3\right)$. The output field shows the result $-\frac{5}{2}$.

Question 31



A screenshot of a CAS interface window titled "Edit Action Interactive". The window contains a toolbar with icons for "0.5 1/2", a hand, "f dx / f dx d", "Simp", "f dx", and a plus sign. Below the toolbar, the input field contains the expression $\text{diff}\left(\frac{36}{1+\sqrt{x}}, x, 1, 4\right)$. The output field shows the result -1 .

Miscellaneous exercise one

Question 1

a

x	$f(x)$	$g f(x)$	
-2	4	5	
-1	1	-1	
0	0	-3	{-3, -1, 5}
1	1	-1	
2	4	-5	

b

x	$f(x)$	$g f(x)$	
-2	-7	49	
-1	-5	25	
0	-3	9	{1, 9, 25, 49}
1	-1	1	
2	1	5	

c

x	$f(x)$	$g f(x)$	
-2	4	2	
-1	1	1	
0	0	0	{0, 1, 2}
1	1	1	
2	4	2	

Question 2

- a** $\frac{dy}{dx}$ is always positive for graphs A and E.
- b** $\frac{dy}{dx}$ is always negative for F.
- c** $\frac{dy}{dx}$ is never negative for A, D and E.
- d** $\frac{dy}{dx}$ is independent of x for A, D and F.

Question 3

$$y = 5 - 7x^2$$

$$\frac{dy}{dx} = -14x$$

$$\frac{d^2y}{dx^2} = -14$$

Question 4

- a** $\frac{dy}{dx} = 10x$
- b** $\frac{dy}{dx} = 10x$
- c** $\frac{dy}{dx} = 2(3 + 5x) \times 5$
 $= 10(3 + 5x)$
 $= 30 + 50x$

Question 5

a $y = (x+1)(x-3)$

$$\begin{aligned}\frac{dy}{dx} &= (x-3) \times 1 + (x+1) \times 1 \\ &= x-3+x+1 \\ &= 2x-2\end{aligned}$$

b $y = (2x-1)(5x+4)$

$$\begin{aligned}\frac{dy}{dx} &= (5x+4) \times 2 + (2x-1) \times 5 \\ &= 10x+8+10x-5 \\ &= 20x+3\end{aligned}$$

c $y = (2x+3)(2x+3)$

$$\begin{aligned}\frac{dy}{dx} &= (2x+3) \times 2 + (2x+3) \times 2 \\ &= 4x+6+4x+6 \\ &= 8x+12\end{aligned}$$

d $y = (x^2-4)(3x+5)$

$$\begin{aligned}\frac{dy}{dx} &= (3x+5) \times 2x + (x^2-4) \times 3 \\ &= 6x^2+10x+3x^2-12 \\ &= 9x^2+10x-12\end{aligned}$$

Question 6

$$y = 2(x^2-5)^7$$

$$\begin{aligned}\frac{dy}{dx} &= 7 \times 2(x^2-5)^6 \times 2x \\ &= 28x(x^2-5)^6\end{aligned}$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= 28(-2) \left[(-2)^2 - 5 \right]^6 \\ &= -56 \times [4-5]^6 \\ &= -56\end{aligned}$$

Question 7

$$y = \frac{x^3 - 3x^2}{x}$$
$$= x^2 - 3x$$

$$\frac{dy}{dx} = 2x - 3$$

Question 8

$$y = \frac{4}{2x+3}$$
$$= 4(2x+3)^{-1}$$

$$\frac{dy}{dx} = 4 \times (-1) \times (2x+3)^{-2} \times 2$$
$$= \frac{-8}{(2x+3)^2}$$

At $x = -1$,

$$\frac{dy}{dx} = \frac{-8}{(2(-1)+3)^2}$$
$$= -8$$

Question 9

$$y = \frac{2x-3}{x+1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x+1) \times 2 - (2x-3) \times 1}{(x+1)^2} \\ &= \frac{2x+2-2x+3}{(x+1)^2} \\ &= \frac{5}{(x+1)^2}\end{aligned}$$

At $x = 3$,

$$\begin{aligned}\frac{dy}{dx} &= \frac{5}{(3+1)^2} \\ &= \frac{5}{16}\end{aligned}$$

Using $\left(3, \frac{3}{4}\right)$

$$\frac{3}{4} = \frac{5}{16}(3) + c$$

$$c = -\frac{3}{16}$$

$$y = \frac{5}{16}x - \frac{3}{16}$$

$$16y = 5x - 3$$

Question 10

$$\frac{13x+1}{2x+2} = x+2 \Rightarrow x = 0.5, 3$$

$$\text{When } x = 0.5, \frac{13x+1}{2x+2} = 2.5$$

$$\text{When } x = 3, \frac{13x+1}{2x+2} = 5$$

The points of intersection are (0.5, 2.5) and (3, 5).

$$\frac{dy}{dx} = \frac{6}{(x+1)^2}$$

When $x = 3$,

$$\begin{aligned} \frac{dy}{dx} &= \frac{6}{(3+1)^2} \\ &= \frac{3}{8} \end{aligned}$$

When $x = 0.5$,

$$\begin{aligned} \frac{dy}{dx} &= \frac{6}{(0.5+1)^2} \\ &= \frac{8}{3} \end{aligned}$$

Question 11

a $y = (x+4)(2x-1)$

$$\begin{aligned} \frac{dy}{dx} &= (x+4) \times 2 + (2x-1) \times 1 \\ &= 2x+8+2x-1 \\ &= 4x+7 \end{aligned}$$

b $y = (3x-1)[(x+4)(2x-1)]$

$$\begin{aligned} \frac{dy}{dx} &= (3x-1)(4x+7) + 3(x+4)(2x-1) \\ &= 12x^2 + 21x - 4x - 7 + 3(2x^2 + 8x - x - 4) \\ &= 12x^2 + 17x - 7 + 6x^2 + 21x - 12 \\ &= 18x^2 + 38x - 19 \end{aligned}$$

The screenshot shows a TI-84 Plus calculator interface. The top menu bar includes 'Edit', 'Action', and 'Interactive'. The main display area shows the equation $\text{solve}\left(\frac{13x+1}{2x+2} = x+2, x\right)$ and the solutions $\{x=3, x=\frac{1}{2}\}$. Below this, the calculator evaluates the function $\frac{13x+1}{2x+2}$ at $x=0.5$ (resulting in $\frac{5}{2}$) and at $x=3$ (resulting in 5). It also shows the derivative $\frac{d}{dx}\left(\frac{13 \cdot x+1}{2 \cdot x+2}\right)$ evaluated at $x=3$ (resulting in $\frac{6}{(x+1)^2}$) and at $x=0.5$ (resulting in $\frac{3}{8}$). The bottom of the screen shows the mode settings: 'Alg', 'Standard', 'Real', 'Deg'.